and the optimum specific impulse I_0 by

$$I_0 = \frac{2(x_0 - l)}{g^2Gb} = \frac{350(x_0 - l)}{b} \text{ sec} \qquad (3)$$

where the second equalities are for b expressed in kilograms per kilowatt and for $G = 0.06$, as for a low-altitude polar orbit. It is also easy to show that the power level is given by

$$P = 2.9 \times 10^{-3} I_0 m_0 \text{ kw} \qquad (4)$$

where $m_0 = m_L/l$ is the initial total mass in kilograms. Typical values of x_0 obtained graphically are $l = 0.1$, $x_0 = 0.46$, $l = 0.5$, $x_0 = 0.72$.

For a payload ratio $l = 0.1$, Eq. (2) may be written $\Delta t_m = (10^{-2}/b)$ days. The corresponding optimum specific impulse from Eq. (3) is $I_0 \approx (125/b)$ sec. It is evident that a specific powerplant mass of about 10^{-2} kg/kw and a specific impulse of about 10^4 sec are required for a duration of even one day. Such a specific powerplant mass is at least two orders of magnitude lower than present design objectives, and one day is too short a time to be of interest for communication satellites, although it might be acceptable for certain special applications. Lower values of $G = \omega v/g$ and, hence, increased duration can be obtained by going to larger circular orbits. However, as v decreases only as the square root of the ratio of the earth's radius to the radius of the orbit, no more than a factor of about 2 can be gained in this way without approaching the 24-hr orbit at six earth radii.

There is no solid basis for speculation as to whether the required specific powerplant masses might become feasible in the future. However, it is worth noting that for sizable payloads the power level given by Eq. (4) is quite high. For example, for a 200 kg payload it approaches 100 mw for $l = 0.1$. In such a power range, radiator mass is predominant, and progress may be largely dependent on radical advances in the field of heat rejection.

Some Energy and Momentum Considerations in the Perforation of Plates

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An early interpretation of experimental data on armor perforation by small metal fragments, moving at ordnance speeds, was made by considering momentum to be conserved between the impacting fragment and the target mass thrown out, leading to the relation

$$v_r/v_s = 1/(1 + \alpha) \qquad (1)$$

where v_r is the fragment residual velocity, v_s the fragment striking velocity, and α the ratio of the target mass thrown out to the fragment mass. The residual velocity of the fragment is assumed to be equal to the ejection velocity of the target pieces. There is good agreement with steel target thicknesses up to about 0.5 cm between this simple expression and the data obtained by Spells.[1] Furthermore, the formula predicts fairly well the data obtained by Jameson and Williams.[2] In general, the experimental data are predicted more closely for small target thicknesses and when v_s is well above the limiting velocity for perforation.

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A similarly elementary but somewhat more rigorous treatment is possible by applying the conservation of energy and momentum to the complete system. We consider the perforation process as a collision between the fragment, the ejected target pieces, and the bulk of the target. On the basis of this model, the following assumptions are made. The energy lost to the target is a constant for a given target. The transfer of momentum to the target pieces is by direct inelastic impact. Impact occurs perpendicular to the surface and at the center of the mass of the target. Furthermore, the pieces emerge with approximately the same velocity, namely, v_r, which is in good agreement with the data, and the momenta of the target pieces are parallel to that of the fragment. Then, the energy and momentum equations, respectively, take the form

$$\tfrac{1}{2}mv_s^2 = \tfrac{1}{2}(m + m_t)v_r^2 + \tfrac{1}{2}(M - m_t)V^2 + W \qquad (2)$$

$$mv_s = (m + m_t)v_r + (M - m_t)V \qquad (3)$$

where m and m_t are, respectively, the masses of the fragment and target pieces, M and V are the mass and velocity of the target, and W is what may be called the work of penetration, which includes shock-wave energy dissipation. In general, W will depend on several variables, including the impact velocity, the properties of the target material, and the properties and shape of the fragment.

Eliminating V between Eqs. (2) and (3) and solving for the ratio v_r/v_s results in

$$\frac{v_r}{v_s} = a + \left[a^2 + \frac{a}{bc}\left(1 - b - \frac{2W}{mv_s^2}\right)\right]^{1/2} \qquad (4)$$

The positive root is chosen since v_r/v_s must be positive on physical grounds. The symbols are defined by

$$a = \mu/(1 + \mu) \qquad b = \mu/(1 + \lambda) \qquad c = 1 + \alpha$$

where $\mu = m/M$ and $\lambda = m_t/M$ are mass ratios. If the target mass M is appreciably larger than m and m_t, then $a \simeq \mu$ and $a/(bc) \simeq (1 - \lambda)/(1 + \alpha)$. Thus, (4) reduces to

$$\frac{v_r}{v_s} = \mu + \left[\mu^2 - \mu + \frac{1 - \lambda}{1 + \alpha}\left(1 - \frac{2W}{mv_s^2}\right)\right]^{1/2}$$

Now, if M is much larger than m and m_t in such a way that $M \to \infty$, then a second approximation, by inspection, results in

$$\frac{v_r}{v_s} = \left[\frac{1}{1 + \alpha}\left(1 - \frac{2W}{mv_s^2}\right)\right]^{1/2} \qquad (5)$$

The latter result is the expression one obtains from the energy equation (2) if the kinetic energy imparted to the target is neglected.

We now separate the inelastic energy loss from W by writing $W = E + E_0$, where E is the energy lost by inelastic impact between the fragment and the target pieces, and E_0 is the energy lost to the rest of the target. If E were the only energy loss, the energy and momentum equations could be written as

$$\tfrac{1}{2}mv_s^2 = \tfrac{1}{2}(m + m_t)\bar{v}_r^2 + E_T + E$$

$$mv_s = (m + m_t)\bar{v}_r + P_T$$

where the kinetic energy and momentum imparted to the target are now denoted by E_T and P_T, respectively. Eliminating $\bar{v}_r$ and solving for E gives, after rearranging,

$$E = \frac{1}{2}\left(\frac{\alpha}{1 + \alpha}\right) mv_s^2 \times \left\{\frac{1 + \alpha}{\alpha}\left(1 - \frac{2E_T}{mv_s^2}\right) - \frac{1}{\alpha}\left(1 - \frac{P_T}{mv_s}\right)^2\right\} \qquad (6)$$

This expression for E involves the quantities $2E_T/(mv_s^2)$ and $P_T/(mv_s)$, the former being the ratio of the kinetic energy of the target to the initial energy of the projectile, and the latter being the ratio of the target momentum to the initial momentum of the projectile. If these ratios are small, they can be neglected and (6) will reduce to

$$E = \tfrac{1}{2}[\alpha/(1 + \alpha)]mv_s^2$$

which is the elementary expression for the energy loss in a completely inelastic collision. Consequently,

$$W = E_0 + \tfrac{1}{2}[\alpha/(1 + \alpha)]mv_s^2 \qquad (7)$$

The substitution of (7) into (5) yields

$$\frac{v_r}{v_s} = \left[\frac{1}{1+\alpha}\left(1 - \frac{\alpha}{1+\alpha} - \frac{2E_0}{mv_s^2}\right)\right]^{1/2} \qquad (8)$$

Finally, if E_0 represents a small part of the impact energy, the term involving E_0 in (8) may be ignored. Such would be the case for thin targets and when v_s is well above the limiting velocity for perforation. Thus, we obtain from Eq. (8),

$$v_r/v_s = 1/(1 + \alpha)$$

which is the simple expression (1).

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Effect of Argon Addition on Shock-Layer Radiance of CO_2-N_2 Gas Mixtures

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1. Effect of Argon Addition

The uncertainties in the chemical composition of the Mars and Venus atmospheres underlying many of the studies of possible unmanned planetary missions can cause an undesirable margin of heat-shield weight to be added, which reduces the available payload and usefulness of these space probes.

Recently, the possibility of relatively large amounts of argon in the Mars atmosphere[1] required a reappraisal of the equilibrium radiative heat transfer to blunt entry bodies for these missions. As a monatomic gas, the presence of argon would be expected generally to raise the temperature and therefore the radiation of the shock layer in front of an entry body for the same atmospheric density and vehicle speed. However, in certain speed ranges, dilution of the basic mixture by argon addition will reduce total radiation from the gas if the argon emissivity itself is low. Thus, there is no clearly defined trend to changes in radiative energy transfer due to the argon addition for all flight velocities of interest.

A Jet Propulsion Laboratory thermochemistry and real-gas normal shock computer program, used previously to give solutions to the equilibrium gas radiance of a CO_2-N_2 mixture,[2] has recently been extended to cases of high argon content. This program used thermochemistry input data obtained from the General Electric Company Missile and Space

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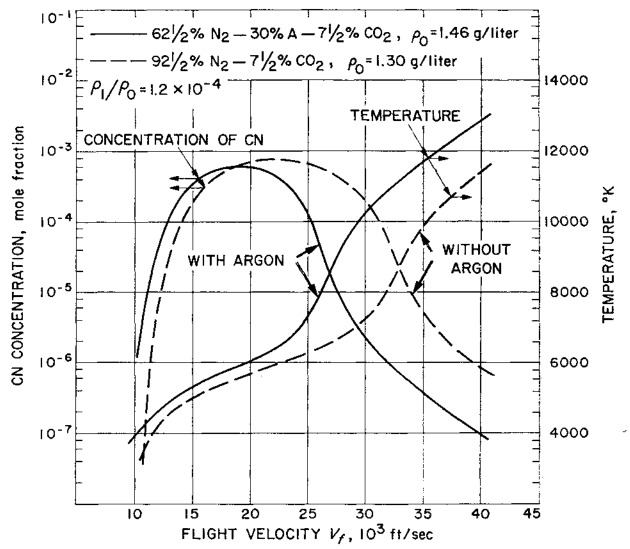

Fig. 1 Temperature and concentration of CN in mixtures with and without argon.

Vehicle Department,[3] and the radiance was computed with existing emissivity data.[4] In the lower speed range, up to 30,000 fps, the molecular band systems of the CN radical contribute the largest part of the total radiated energy for a CO_2-N_2 mixture. This was first suggested in Ref. 5 and shown in Ref. 6. With argon added, the present calculations show that the CN concentration in the gas mixture is affected such as to reduce the number of CN particles formed and to shift the peak in the concentration to lower flight velocities (Fig. 1). This reduced concentration, more than compensating for generally higher temperatures, causes the radiance of the argon mixture to fall below the comparable value for a CO_2-N_2 mixture (Fig. 2) in the relatively narrow speed range between 21,000 and 27,000 fps. Up to the latter speed, the effect of argon addition is relatively small and of varying direction compared to the higher speed range. For higher speeds, the main radiation mechanism is deionization and electron ac-

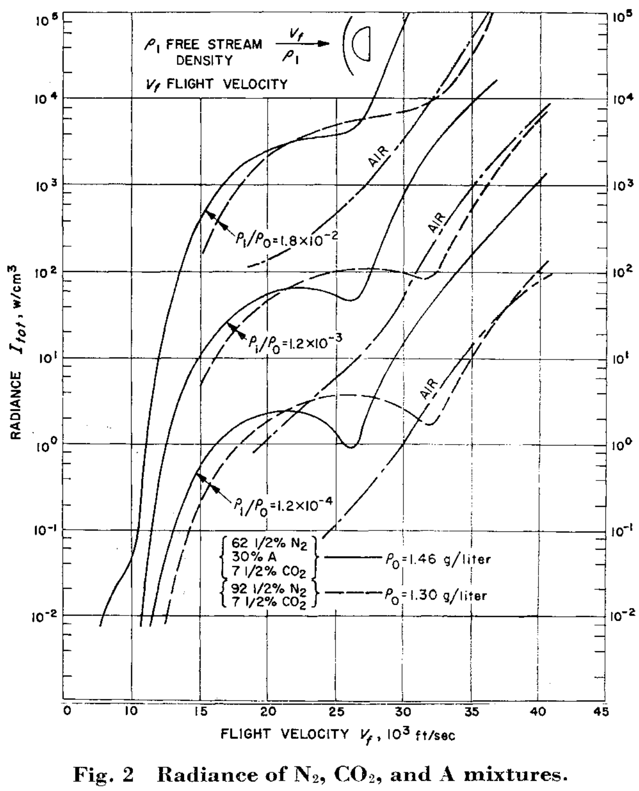

Fig. 2 Radiance of N_2, CO_2, and A mixtures.